Gravity model of trade as a representative of the ensemble of maximally random networks

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Cooking and physics

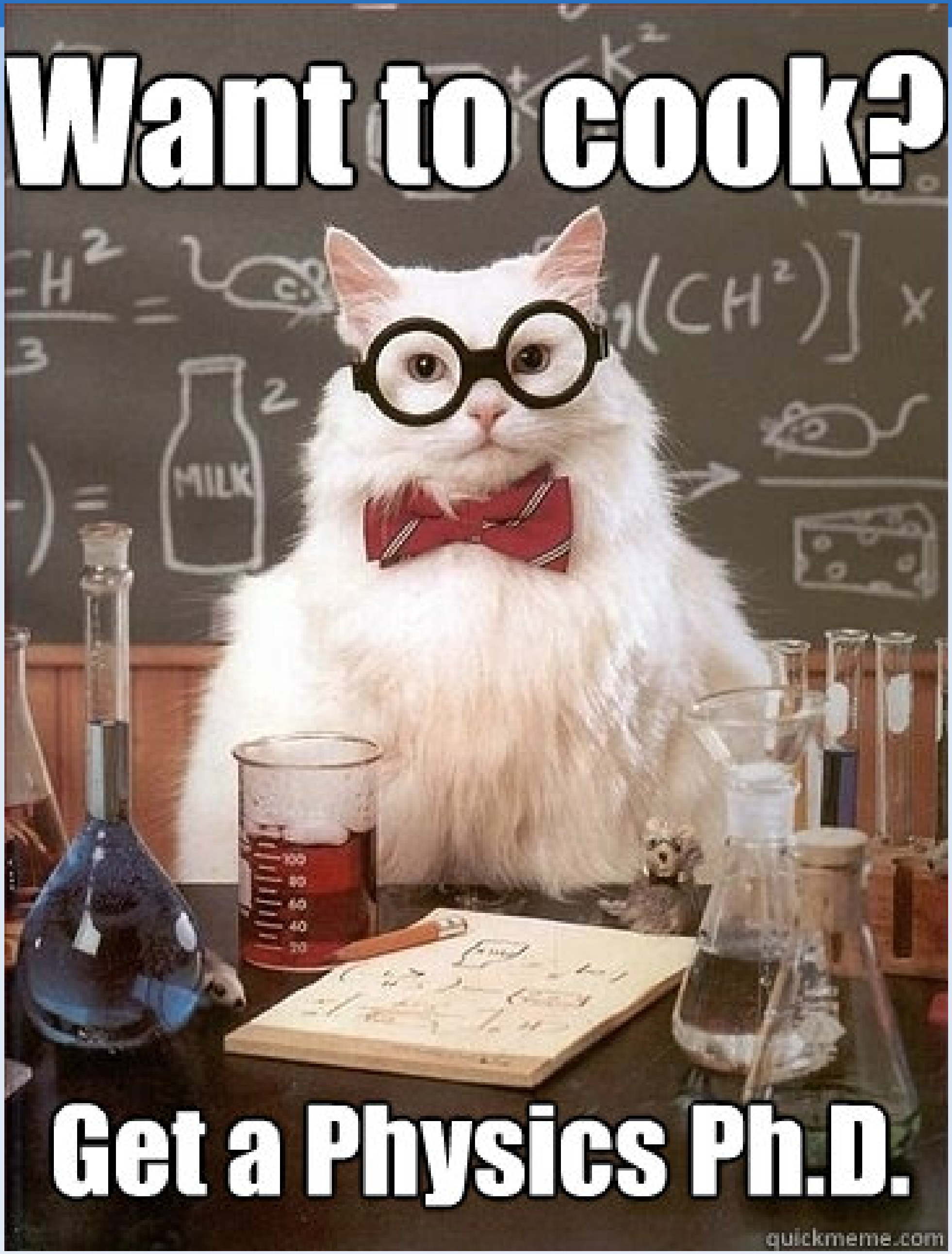
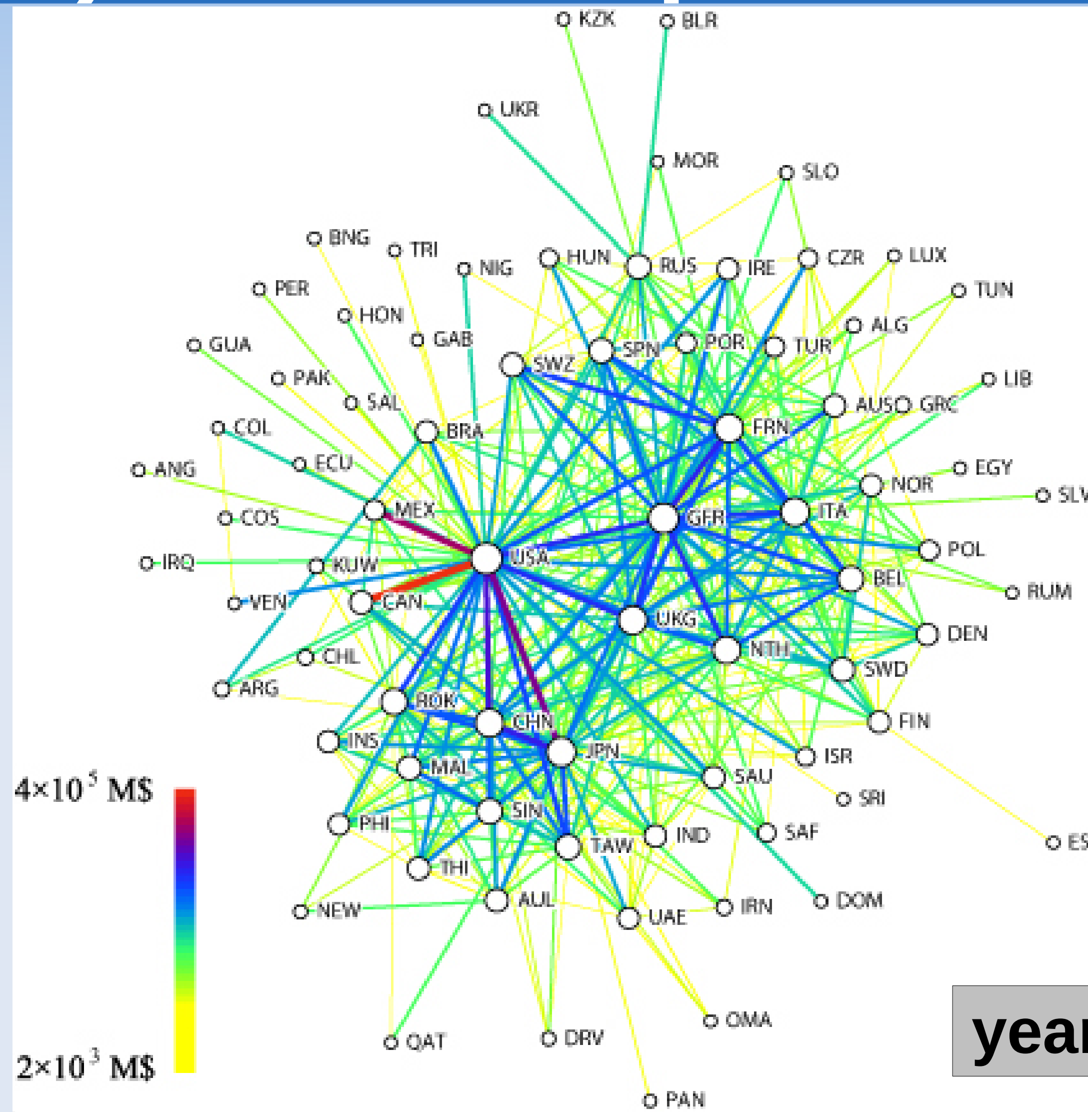


Image source: http://www.quickmeme.com

The International Trade Network (ITN) – an example

Image source: http://www.intennar.com/2012/06/11/the-5-essential-ingredients-for-killer-sales-and-marketing-content/



year 2000

Source: Bhattacharya K.; Mukherjee G.; Saramäki J.; Kaski K.; Manna S. S. The International Trade Network: weighted network analysis and modelling. *J. Stat. Mech*., P02002, 2008.

Countries of ITN

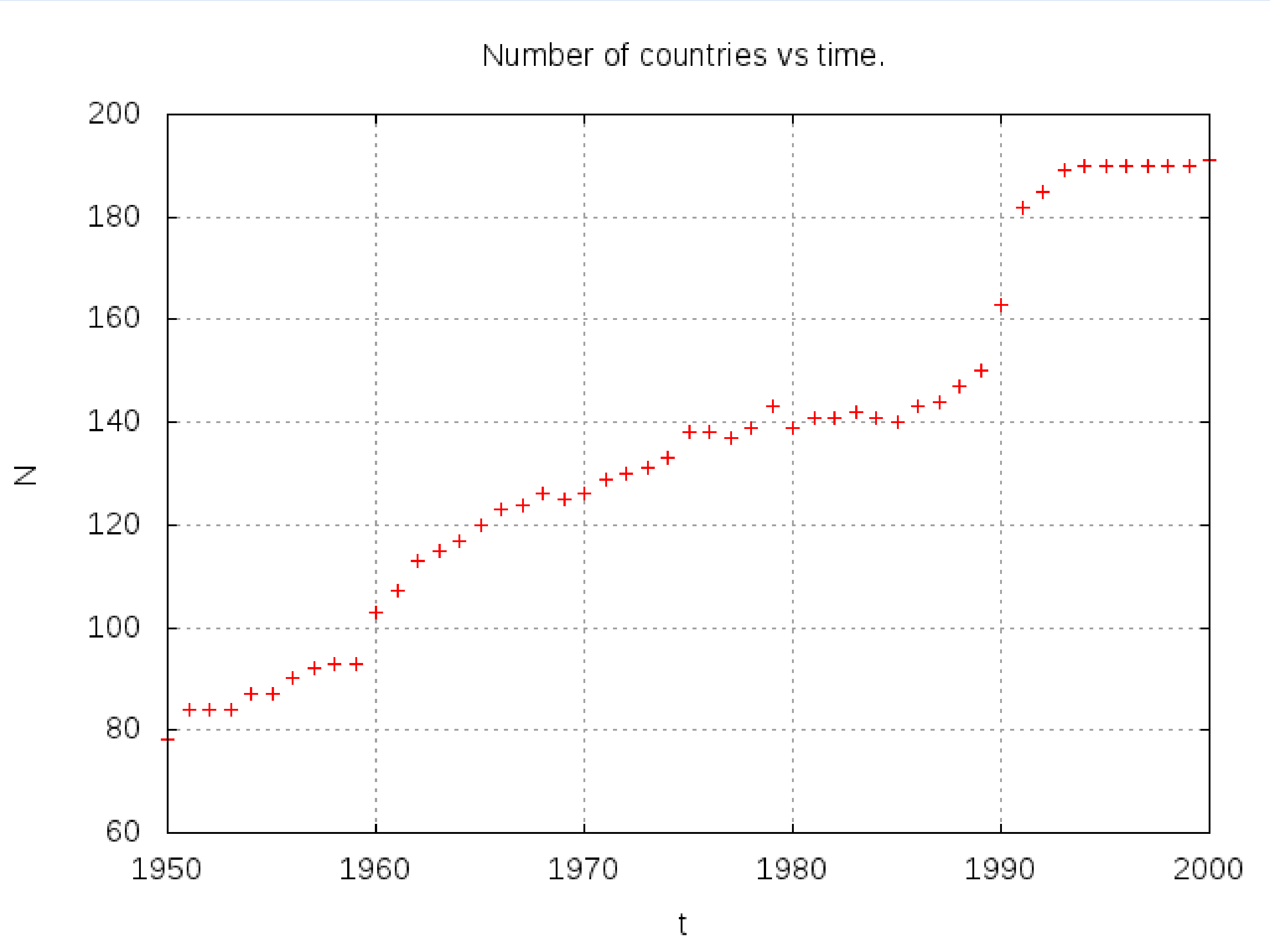


Image source: http://www.simplyhelpinghim.com/2013/05/06/4091/

Gravity model of trade

Called “gravity model” for its analogy with Newton’s law of Universal Gravitation.

- Newton’s Law of Universal Gravitation

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2}$$

F = attractive force; **M** = mass;

D = distance; **G** = gravitational constant

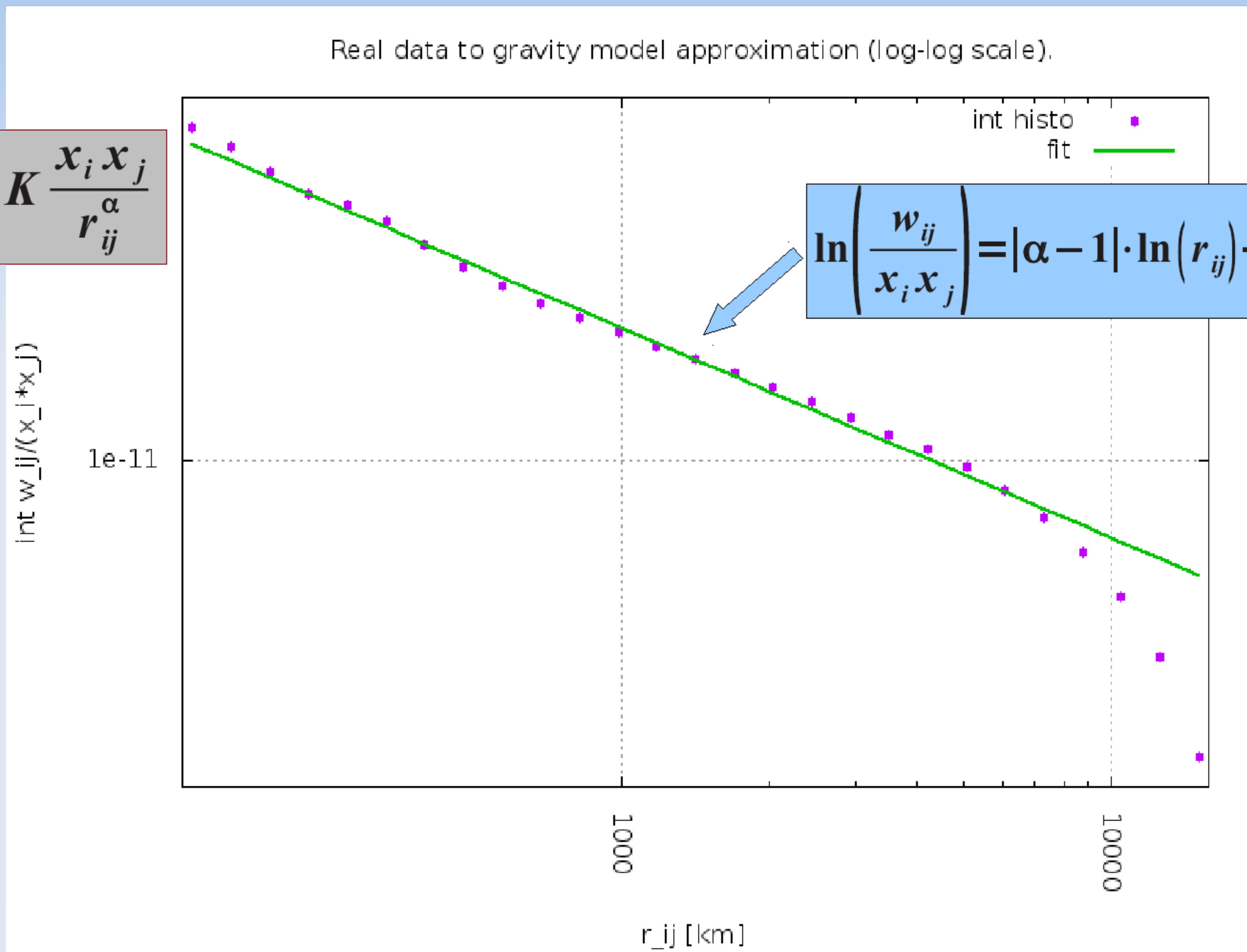
- Gravity model of trade

$$w_{ij} = K \frac{x_i x_j}{r_{ij}^{\alpha}}$$

$\mathbf{w}_{ij}$ = trade volume (export/import) from i to j; **x** = economic size (i.e. GDP); **r** = geographic distance; **K** = trade constant

Including gravity model

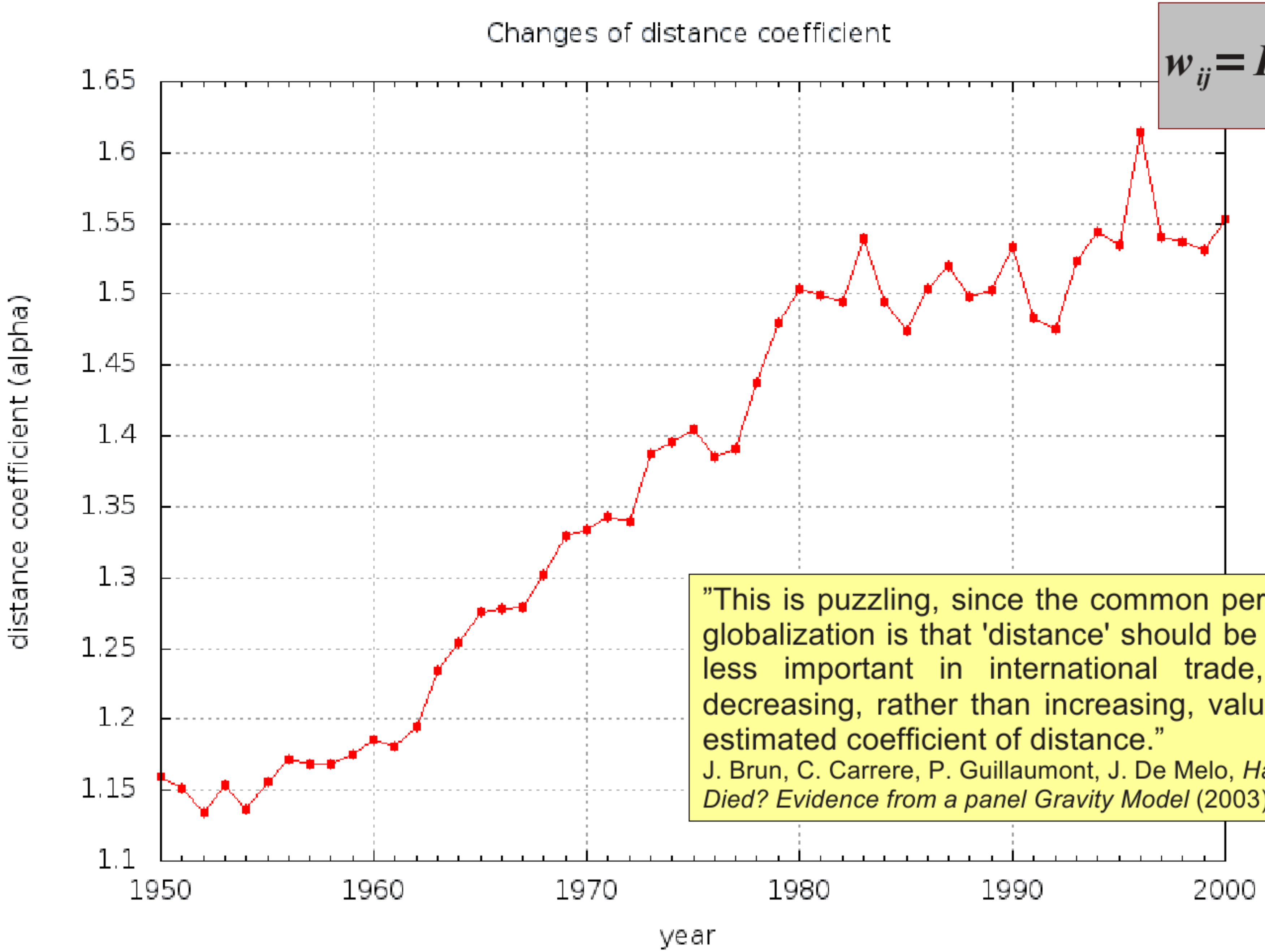
$$w_{ij} = K \frac{x_i x_j}{r_{ij}^{\alpha}}$$



$$\ln\left(\frac{w_{ij}}{x_i x_j}\right) = |\alpha - 1| \cdot \ln(r_{ij}) + b$$

Growth of alpha coefficient

$$w_{ij} = K \frac{x_i x_j}{r_{ij}^{\alpha}}$$



”This is puzzling, since the common perception of globalization is that 'distance' should be becoming less important in international trade, implying decreasing, rather than increasing, values for the estimated coefficient of distance.”

J. Brun, C. Carrere, P. Guillaumont, J. De Melo, *Has Distance Died? Evidence from a panel Gravity Model* (2003)

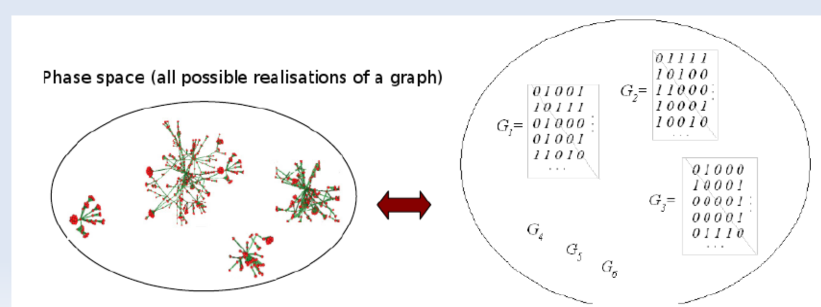
Image source: http://ericaskitchenadventures.blogspot.com/2009_11_01_archive.html

Exponential random graphs

- Specify a set of networks $\mathcal{G}$ = {G},
- Decide what constraints should be imposed on the ensemble (e.g. properties of real network),
- Maximalize Gibbs-Shannon entropy:

$$S = -\sum_{\wp \in G} P(G) \ln P(G)$$

- P(G) will be probability distribution associated with given constraints.



Exponential random graphs

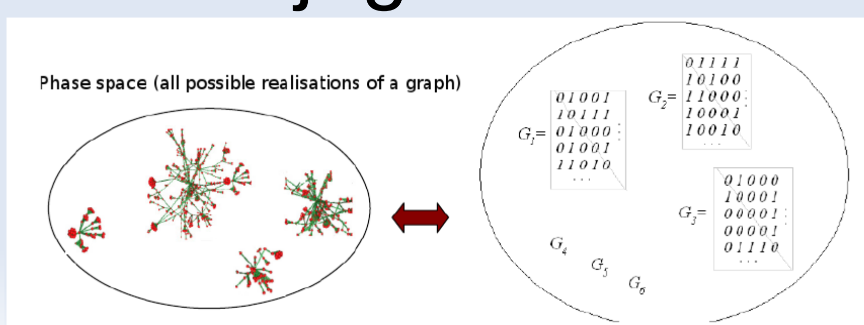
$$P(G) = \frac{e^{-H(G)}}{Z} \qquad \text{where} \qquad Z = \sum_{\wp \in G} e^{-H(G)}$$

In general:

$$H(G) = \sum_i \theta_i A_i(G)$$

$\{A_i(G)\}$ - set of parameters of the ensemble
(i.e. graph observables, eg. structural properties)

$\{\theta_i\}$ - set of fields conjugated to these parameters



Let's stir it!

Image source: http://crockpot365.blogspot.com/2013/04/5-ingredient-homemade-beef-stew.html

Improved hamiltonian of ITN (with distances)

Ensemble of directed weighted networks, which is described by Hamiltonian (for ”local trade” only):

$$H(G) = \sum_i \sum_{j \neq i} \theta_{ij} w_{ij}$$

where $\theta_{ij} = \frac{B}{T} \frac{r_{ij}^{\alpha}}{x_i x_j}$ are Lagrange multipliers and

w_{ij} - value of import/export between country *i* and *j*

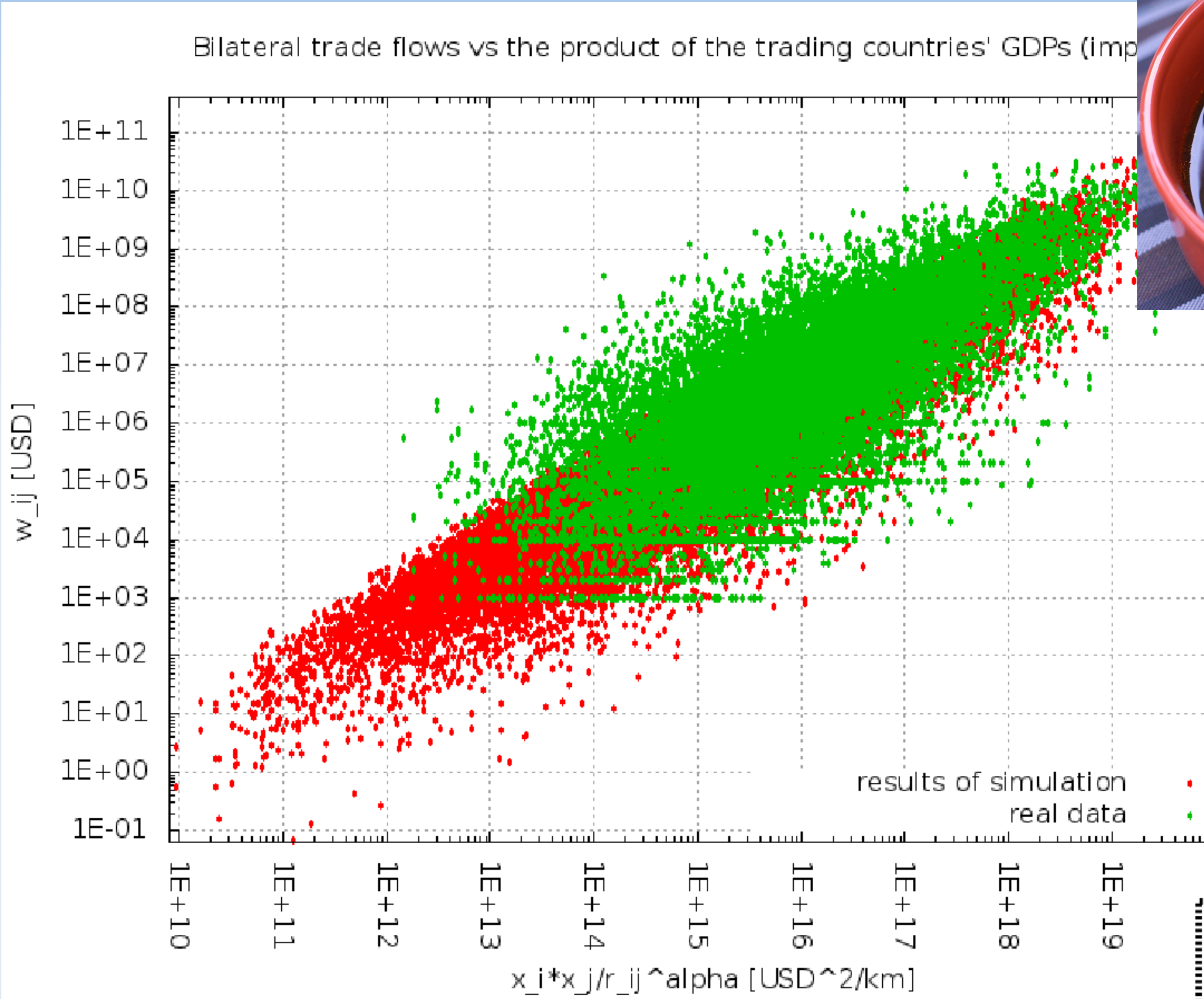
T - total trade $T = \sum_i \sum_{j \neq i} w_{ij}$

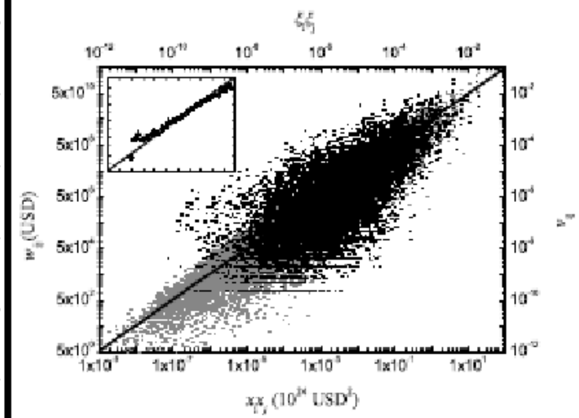
$$B = \sum_i \sum_{j \neq i} x_i \cdot x_j / r_{ij}^{\alpha}$$

x_i, x_j - GDPs of countries *i* and *j*

r_{ij} - distance between capital cities

Modelling of ITN (with distances) simulation





Bilateral trade flows in 1995 vs. the product of the trading countries' GDPs (points) as compared with their theoretical prediction.

Source: A. Fronczak, P. Fronczak, *Statistical mechanics of the international trade network*, Phys. Rev. E 85, 056113 (2012)

Our motivation

Analysis of changes of ITN year-by-year will fuel future works and may answer question:

Is it possible to predict crysis that appears in ITN?

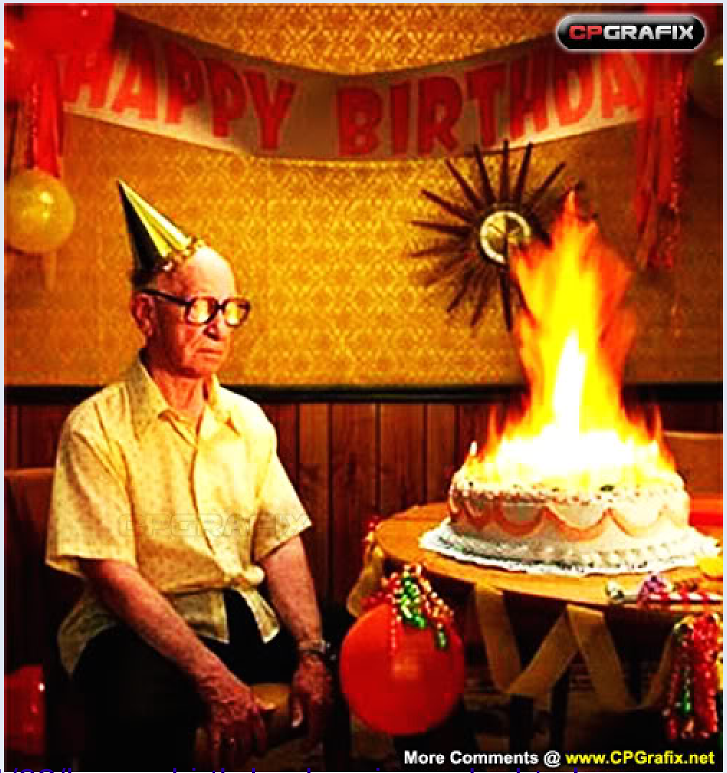


Image source: http://www.cpgrafix.org/2011/02/happy-birthday-burning-cake.html

Fluctuation-response theory

From the first version of the model:

$$\frac{d\langle v_{ij}\rangle}{\langle v_{ij}\rangle}=\frac{d\xi_i}{\xi_i}+\frac{d\xi_j}{\xi_j},$$

where

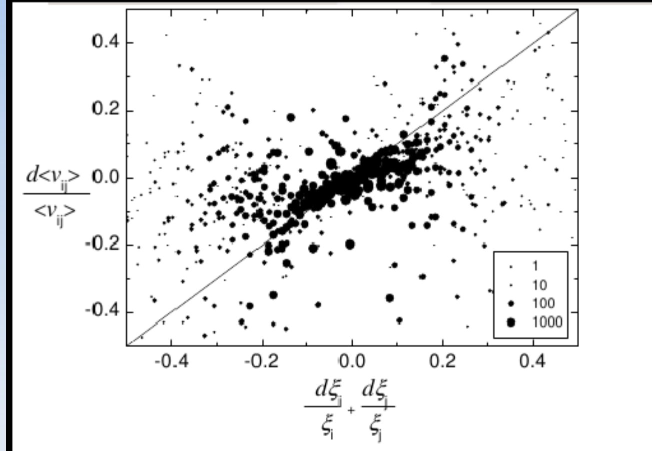
$$\langle v_{ij}\rangle=\langle w_{ij}\rangle/\sum_{i,j}\langle w_{ij}\rangle,\quad \xi_i=x_i/\sum_i \xi_i,\quad \xi_j=x_j/\sum_j \xi_j$$

After adding distances:

$$\frac{d\langle v_{ij}\rangle}{\langle v_{ij}\rangle}=\frac{d\xi_i}{\xi_i}+\frac{d\xi_j}{\xi_j}-\ln\left(\frac{r_{ij}}{R}\right)\cdot d\alpha,$$

where

$$R=[\sum_{i,j}(r_{ij}/(\xi_i\xi_j)^{-\alpha})]^{-1}$$



Fluctuation-response theorem for ITN

Source: A. Fronczak, P. Fronczak, *Statistical mechanics of the international trade network*, Phys. Rev. E 85, 056113 (2012)

Thank you for your attention!

References:

- A. Fronczak, P. Fronczak, Statistical mechanics of the international trade network, Phys. Rev. E 85, 056113 (2012)
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- M. A. Serrano, M. Boguñá, and A. Vespignani. Patterns of dominant flows in the world trade web, Journal of Economic Interaction and Coordination 2, 111-124 (2007)

Image source: http://en.wikipedia.org/wiki/Chef_%28South_Park%29